Int. J. Adv. Appl. Math. and Mech. 9(2) (2021) 38 – 45 (ISSN: 2347-2529)
Journal homepage: www.ijaamm.com

IJAAMM

International Journal of Advances in Applied Mathematics and Mechanics

Long time solutions for a coupled parabolic and Hamilton-Jacobi equations

Research Article

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Received 18 November 2021; accepted (in revised version) 16 December 2021

Abstract: This work is concerned with the long time solutions for a reaction-diffusion equation coupled with an Hamilton-Jacobi equation describing the propagation of a flame front in a non homogeneous medium. We present here an analytical formulation of the change of the temperature profile during the combustion.

MSC: 35B40 • 35M11

Keywords: Striated medium • Stationnary solution • Viscosity solution • Hump effect

1. Introduction

The study of this problem began since ([1], [2], [3]) where existence and uniqueness theorems have been proved according to the different configurations due to the heterogeneity of the material.

In this work, we deal with the change in the temperature profile observed in the horizontal case ($\alpha = \pi/2$). This case seems to behave like the limit for $\alpha \in [0\,,\,\pi/2\,[$. For example, the periodicity property is no longer verified in the direction of the propagation where the solution becomes stationary.

This change in profile was observed in [2] and [3]. In the present paper, we characterize this change by giving an analytical formulation of the stationary solution at the front.

We then have to solve the problem below:

$$P_u \begin{cases} \dfrac{\partial u}{\partial t} + \mathscr{A}\, u = 0 & in \ \Omega_0^T \\ u(x_1, x_2, 0) = u_0(x_1, x_2) & in \ \Omega_0 \end{cases} \tag{1}$$

with $\mathscr{A} = a_0 \dfrac{\partial}{\partial x_1} - \dfrac{\partial}{\partial x_i}\left[a_{ij} \dfrac{\partial}{\partial x_j} \right]$ where $u = u(x_1 + \xi, x_2, t)$ is the temperature in Ω_0 at the time t.

We note $x = (x_1,\ x_2)$ and $a_{11} = \left[1 + \left(\dfrac{\partial \xi}{\partial x_2} \right)^2 \right] d(x)$, $a_{12} = a_{21} = -\dfrac{\partial \xi}{\partial x_2} d(x)$,

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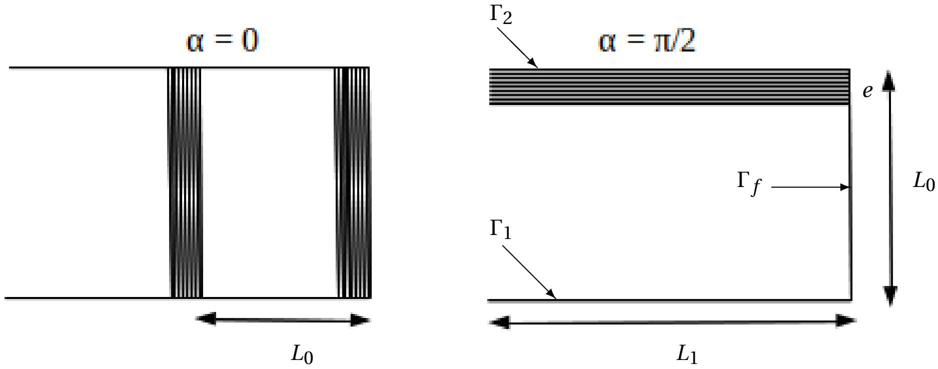

Fig. 1. *Domain* Ω_0 *configurations*

$a_{22} = d(x), a_0 = -\frac{\partial \xi}{\partial t}$. Let $L_0 > 0$ be the length of one period (cell) of the domain like in Figure 1.
We note $\Omega_0 =]-L_1, 0[\times]0, L_0[$ with $L_1 >> L_0$ (to ensure $u_{\Gamma_0} = 0$), $\Omega_0^T = \Omega_0\times]0, T[$, $Y =]0, L_0[$, $Y^T =]0, L_0[\times]0, T[$. The boundary $\partial\Omega = \Gamma = \Gamma_0 \cup \Gamma_1 \cup \Gamma_2 \cup \Gamma_f$ whith

$$\Gamma_0 = \{x_1 \equiv -L_1\}, \quad \Gamma_1 = \{x_2 \equiv 0\}, \quad \Gamma_2 = \{x_2 \equiv L_0\}, \quad \Gamma_f = \{x_1 \equiv 0\}$$

The conditions on Γ are:

$$a_{11}\frac{\partial u}{\partial x_1} - a_{12}\frac{\partial u}{\partial x_2} = -\theta_0\frac{d\xi}{dt} \text{ on } \Gamma_f, \ u_{\Gamma_0} = 0, \ u_{\Gamma_1} = u_{\Gamma_2} \text{ and } \frac{\partial u}{\partial x_2}\Gamma_1 = \frac{\partial u}{\partial x_2}\Gamma_2.$$

$R_0 = R_0(x_1, x_2)$ (the rate of the reaction) and $d = d(x_1, x_2)$ (the diffusivity of the material) are regular, positive, bounded, periodic in x_1 or x_2 with period L_0.
By definition we have $0 < \theta_0 \leq 1$; it is the fraction of energy which contribute to maintain the combustion.
The function $\xi = \xi(x_2, t)$ describes the propagation of the front flame and verifies:

$$P_\xi \begin{cases} \frac{\partial \xi}{\partial t} + R_0(x_2)\sqrt{1 + \left(\frac{\partial \xi}{\partial x_2}\right)^2} = 0 & in \ Y^T \\ \xi(x_2, 0) = \xi_0(x_2) & t = 0 \end{cases} \tag{2}$$

The system $P_{u,\xi} = (P_u, P_\xi)$ models the "Hump effect" which is an anomaly of overvelocity (or overpressure) observed during the combustion of some solid propellant blocks (see [2]) in the combustion room.

Vertical case

When $\alpha = 0$, $P_{u,\xi}$ corresponds to a one dimensional problem because of the non dependance of d and R in x_2. The flame front can be reduced to a point.
Changing the initial condition to a periodic condition for u, we have:

$$P_{u,\xi}^V \begin{cases} P_u^V \begin{cases} \frac{\partial u}{\partial t} - \frac{d\xi}{dt}\frac{\partial u}{\partial x_1} - \frac{\partial}{\partial x_1}\left[d(x_1+\xi)\frac{\partial u}{\partial x_1}\right] = 0 & x_1 < 0, \ t > 0 \\ d(x_1+\xi)\frac{\partial u}{\partial x_1} = \theta_0 R_0(\xi) & x_1 = 0 \\ u \longrightarrow 0 & x_1 \to -\infty \\ u(0) = u(T) \end{cases} \\ \\ P_\xi^V \begin{cases} \frac{d\xi}{dt} = -R_0(\xi) & t > 0 \\ \xi(0) = \xi_0 \end{cases} \end{cases} \tag{3}$$

Under the following hypothesis:

H1) R_0 and d are regular and periodic functions with period L_0 and:

$$0 < \min_{x\in\mathbf{R}} R_0(x) = R_{0l} \leq R_0(x) \leq R_{0c} = \max_{x\in\mathbf{R}} R_0(x) \quad \forall x \in \mathbf{R}$$

$$0 < \min_{x\in\mathbf{R}} d(x) = d_l \leq d(x) \leq d_c = \max_{x\in\mathbf{R}} d(x) \quad \forall x \in \mathbf{R}$$

we have the theorem below (see [2], [4], [5]):

Theorem 1.1.
$P^V_{u,\xi}$ admits a unique solution (u,ξ) where:
$u(x_1,t)\geq 0\ ,\ \ \xi(t)<0\ ,\ \ \forall x_1\leq 0,\ 0<t<T$
$u\in L^2\left[0,T;H^1(\mathbf{R}^*_-)\right]\cap L^\infty\left[0,T;L^2(\mathbf{R}^*_-)\right]$
$\xi\in W^{1,\infty}(0,T), \forall t\in \mathbf{R}^*_+$
u and $\frac{d\xi}{dt}$ are periodic in time with period T^* verifying: $T^*=L_0/\int_0^1 R_0(\xi)d\tau$ and $\frac{L_0}{R_{0c}}\leq T^*\leq \frac{L_0}{R_{0l}}$ where L_0 is the length of the spatial period i.e the sum of the thickness of the linner and the charge.

Proposition 1.1.
The positive real T^* defined in theorem 1.1 verifies:

$$\xi(t+T^*)-\xi(t)=-L_0 \qquad \forall t\geq 0$$

T^* is the time necessary for the front to cover the spatial period L_0.

The numerical results below confirm what is announced.

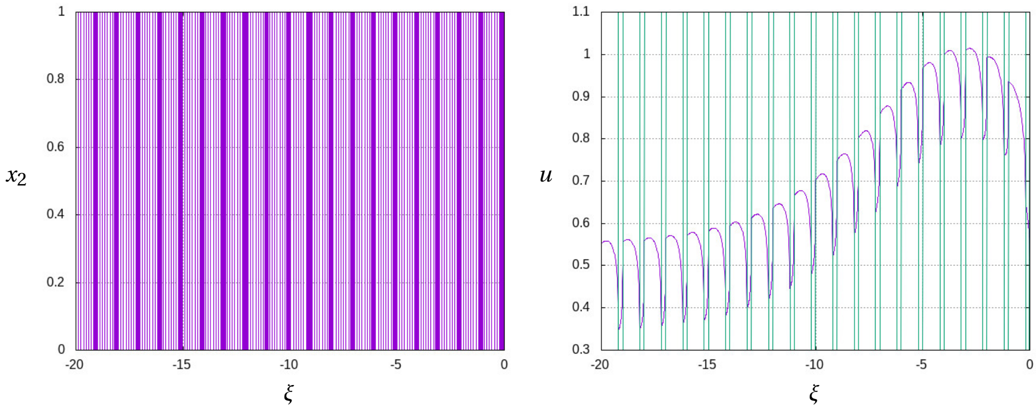

Fig. 2. *Position of the front and Temperature at a point of the front in time.*

Remark 1.1.
We can notice that the speed of the front is periodic and the temperature at a point of the front (it is the same for all the other points of the front), after an unsteady phase, stabilizes, with a periodic profile.

Horizontal case

Still under the hypothesis $H1$ relatively to x_2, we have (see [4], [7], [6], [8]):

Theorem 1.2.
If $\xi_0\in C(]0,L_2[)$, then the problem P_ξ has a unique viscosity solution $\xi\in C(]0,L_0[\times]0,T[)$ i.e satisfying: if (x_0,t_0) is a local maximum (resp minimum) point of $\xi-w$, then $\frac{\partial w}{\partial t}(x_0,t_0)+R_0(x_0)\sqrt{1+(\nabla w(x_0,t_0))^2}\leq 0$ $(resp\geq 0)$. In addition, we have the following inequalities: if $\xi_0\in W^{1,\infty}(\mathbf{R})$, then the viscosity solution $\xi\in W^{1,\infty}(\mathbf{R}\times]0,T[)$ verifies:

$$\|\frac{\partial\xi}{\partial t}\|_{L^\infty(\mathbf{R}\times]0,+\infty[)}\leq c_1 \quad \text{and} \quad \|\frac{\partial\xi}{\partial x_2}\|_{L^\infty(\mathbf{R}\times]0,+\infty[)}\leq c_2$$

where c_1 and c_2 are constants depending only on $\nabla\xi_0$.

Theorem 1.3.
The viscosity solution of P_ξ is periodic in x_2 with period L_0 as soon as ξ_0 and R_0 are periodic with the same period. In addition, ξ converges asymptotically in time to the wave solution $\tilde{\xi}$ of the form:

$$\tilde{\xi}(x_2,t)=-R_{0c}.t+\int_0^{x_2}\tilde{\psi}(x)dx \tag{4}$$

where $\tilde{\psi}$ is the entropic solution of $\dfrac{\partial \psi}{\partial t} + \dfrac{\partial}{\partial x_2}\left(R_0(x_2)\sqrt{1+\psi^2}\right) = 0$ *and* $R_{0c} = \max\limits_{x \in \mathbf{R}R} R_0(x)$.

Note $\mathscr{W} = \left\{u \in H^1(\Omega_0);\ \ u|_{\Gamma_1} = u|_{\Gamma_2},\ \dfrac{\partial u}{\partial x_2}|_{\Gamma_1} = \dfrac{\partial u}{\partial x_2}|_{\Gamma_2},\ \ u|_{\Gamma_0} = 0.\right\}$ provided with the classical H^1 norm.

Theorem 1.4.
P_u admits a unique solution $u \in L^2(0,T;\mathscr{W}) \cap C\left(0,T;L^2\right)$, $\dfrac{du}{dt} \in L^2\left(0,T;\mathscr{W}\,'\right)$ where $\mathscr{W}\,'$ is the dual of $\mathscr{W}$.

Here also the numerical results confirm the theoretical properties.

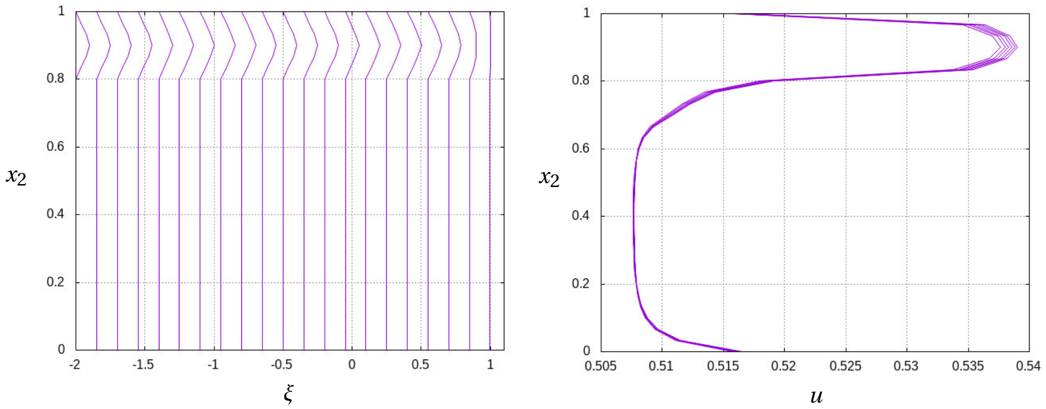

Fig. 3. *Position of the front and Temperature at the front in time.*

Nevertheless, we observe that the profile of the temperature at the front changed.
As in the vertical case, it was expected to have a higher temperature in the "charge" than in the "linner" "charge". Indeed; this is what happens when the combustion starts. After the transional phase, we notice a change as in the figures below.

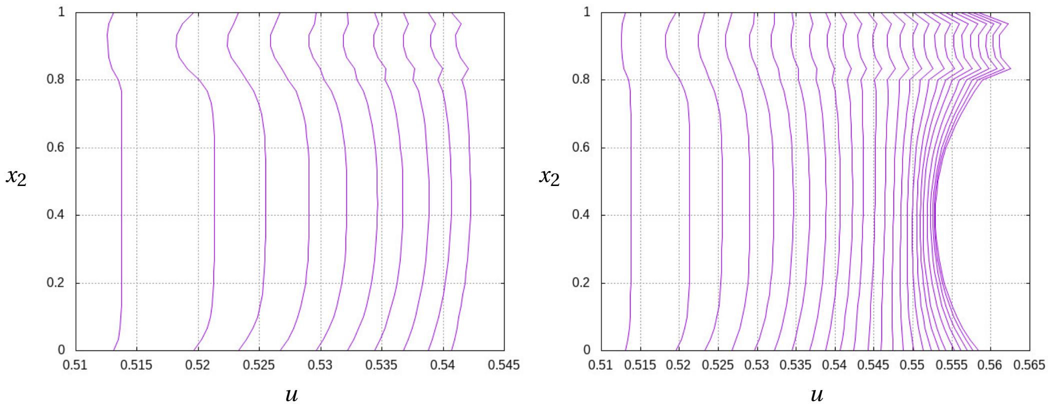

Fig. 4. *Evolution of the temperature profile at the front.*

To understand this phenomenon, we study the case the medium is homogeneous.

Homogeneous Medium

In this case, the corresponding problem $P_{u,\xi}$ is like in equation (3) where the functions d and R_0 are constants. It admits a solution (u,ξ) where $u = u(x_1, t)$ converges asymptotically in time to:

$$\tilde{u}(x_1) = \theta_0 \exp\left\{\frac{R_0}{d} x_1\right\} \qquad \forall x_1 \le 0. \tag{5}$$

So the temperature at the front $u_f = \theta_0$ depends only on θ_0 and the velocity of the flame front becomes $V_f = -R_0$. Inside the domain, the temperature depend on the ratio $\dfrac{R_0}{d}$. We also notice that the greater d is the higher the temperature in the domain. We now present the main result of this paper.

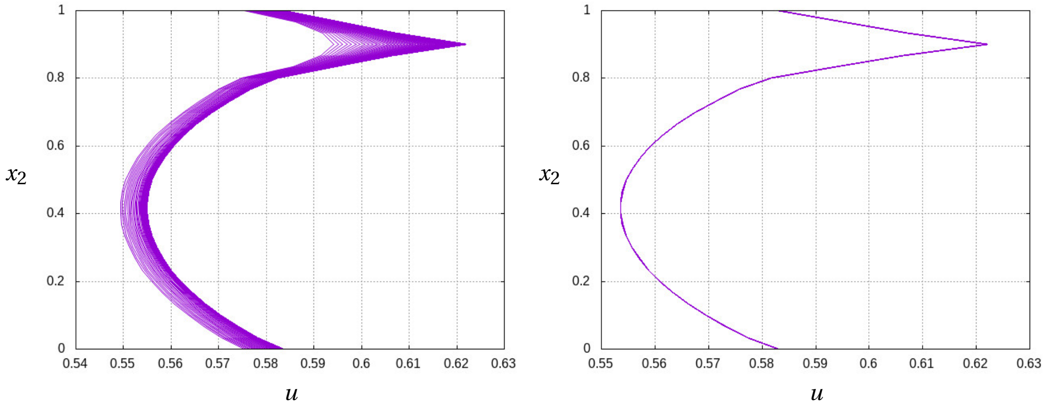

Fig. 5. *Final profile of the temperature at the front.*

Stationary solution in the horizontal case

The stationary problem is as follow:

$$P_{\tilde u}^{Stat}\begin{cases} -a_0\dfrac{\partial \tilde u}{\partial x_1}-\dfrac{\partial}{\partial x_i}\left[a_{ij}\dfrac{\partial \tilde u}{\partial x_j}\right]=0 & (x_1,x_2)\in\Omega_0\\ a_{1j}\dfrac{\partial \tilde u}{\partial x_j}=-\theta_0 R_{0c} & x_1=0\\ \tilde u=0 & x_1=-L_1\\ \tilde u(x_1,0)=\tilde u(x_1,L_0) & x_1\le 0,\\ \dfrac{\partial \tilde u}{\partial x_2}(x_1,0)=\dfrac{\partial \tilde u}{\partial x_2}(x_1,L_0) & x_1\le 0\end{cases} \tag{6}$$

with $a_0 = R_{0c}$ (see equation (4)), $a_{11} = d\left(1+\left(\dfrac{\partial\xi}{\partial x_2}\right)^2\right)$, $a_{12} = a_{21} = -d\dfrac{\partial\xi}{\partial x_2}$, $a_{22} = d$. From the theorem 1.2, we have $a_{ij}(x_2,t)\in L^\infty(]0,L_2[\times]0,+\infty[)$ for $i,\ j=1,\ 2$. So the operator $\mathscr{A}=a_0\dfrac{\partial}{\partial x_1}-\dfrac{\partial}{\partial x_i}\left[a_{ij}\dfrac{\partial}{\partial x_j}\right]$ is elliptic and we have (see [9]):

Theorem 1.5.
$\exists!\tilde u\in L^2(W)\cap C(L^2)$ periodic in x_2 and verifying $P_{\tilde u}^{Stat}$.
$\tilde u$ is the asymptotic solution of P_u.

The equation at the front

In this section, without loss of generality, we set $d(x)=d_c$, $R_0(x)=R_{0c}$ in the *"charge"* and $d(x)=d_l$, $R_0(x)=R_{0l}$ in the *"linner"* (see [10]). So in the *"charge"* we have:

$$d_c\left(1+\left(\frac{\partial\xi}{\partial x_2}\right)^2\right)\frac{\partial u}{\partial x_1}-d_c\frac{\partial\xi}{\partial x_2}\frac{\partial u}{\partial x_2}=\theta_0R_{0c}\,,\quad x_2\in\mathbf{R},\quad t>0 \tag{7}$$

We also have $\dfrac{\partial\xi}{\partial x_2}\longrightarrow 0$ in the charge (see Figure 3) and combining with the equation in the domain, we get:

$$R_{0c}\frac{\partial u}{\partial x_1}-d_c\left(\frac{\partial^2 u}{\partial x_1^2}+\frac{\partial^2 u}{\partial x_2^2}\right)=0 \tag{8}$$

$$d_c\frac{\partial u}{\partial x_1}=\theta_0R_{0c} \tag{9}$$

By substituting (9) in (8), we have:

$$\theta_0\frac{R_{0c}^2}{d_c^2}-\frac{\partial^2 u}{\partial x_2^2}=0 \tag{10}$$

The ordinary differential equation (10) has for solution:

$$u(x_2)=\frac{1}{2}\theta_0\frac{R_{0c}^2}{d_c^2}x_2^2+C_1x_2+C_2 \tag{11}$$

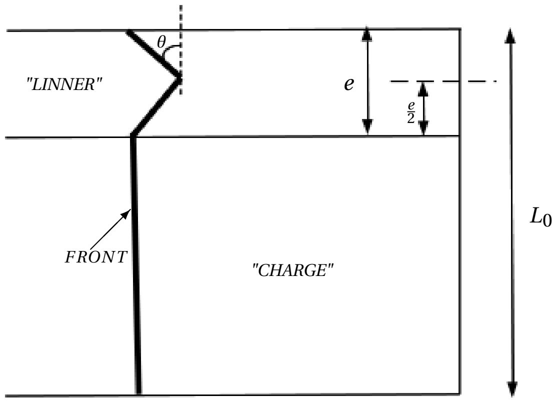

Fig. 6. Periodic front

We notice that u, at the front, has a parabolic profile in the *"charge"*.

In the *"liner"* we have:

$$d_l\left(1+\left(\frac{\partial \xi}{\partial x_2}\right)^2\right)\frac{\partial u}{\partial x_1}-d_l\frac{\partial \xi}{\partial x_2}\frac{\partial u}{\partial x_2}=\theta_0 R_{0c}\,,\quad x_2\in\mathbf{R} \tag{12}$$

$$\frac{\partial \xi}{\partial x_2}\longrightarrow \tan\theta \ \text{ if } x_2\in\left[L_0-e;L_0-\frac{e}{2}\right] \ \text{ and } \ \frac{\partial \xi}{\partial x_2}\longrightarrow -\tan\theta \ \text{ if } x_2\in\left[L_0-\frac{e}{2};L_0\right]$$

where $\theta = arctg\left[\left(-R_{0c}^2+\sqrt{-R_{0l}^4+2R_{0l}^2R_{0c}^2}\right)/\left(R_{0l}^2-R_{0c}^2\right)\right]$ (see [2]).

- If $x_2\in\left[L_0-e;L_0-\frac{e}{2}\right]$ then $\frac{\partial \xi}{\partial x_2}=\tan\theta$ and we get:

$$\begin{cases} R_{0c}\dfrac{\partial u}{\partial x_1}-d_l\left(1+\tan^2\theta\right)\dfrac{\partial^2 u}{\partial x_1^2}+d_l\tan\theta\dfrac{\partial^2 u}{\partial x_1\partial x_2}+ \\ +d_l\tan\theta\dfrac{\partial^2 u}{\partial x_2\partial x_1}-d_l\dfrac{\partial^2 u}{\partial x_2^2}=0 \\ \\ d_l\left(1+\tan^2\theta\right)\dfrac{\partial u}{\partial x_1}-d_l\tan\theta\dfrac{\partial u}{\partial x_2}=\theta_0R_{0c} \end{cases} \tag{13}$$

With the second equation of (13), after derivation with respect to x_1, we have:

$$d_l\left(1+\tan^2\theta\right)\frac{\partial^2 u}{\partial x^2}-d_l\tan\theta\frac{\partial^2 u}{\partial x_1\partial x_2}=0 \tag{14}$$

By introducing it into the first equation of the system (13), with the same first equation from which its derives, we have:

$$R_{0c}\left[\frac{1}{d_l\left(1+\tan^2\theta\right)}\left(\theta_0R_{0c}+d_l\tan\theta\frac{\partial u}{\partial x_2}\right)\right]-d_l\tan\theta\frac{\partial^2 u}{\partial x_1\partial x_2}+d_l\tan\theta\frac{\partial^2 u}{\partial x_1\partial x_2}$$
$$+d_l\tan\theta\frac{\partial}{\partial x_2}\left[\frac{1}{d_l\left(1+\tan^2\theta\right)}\left(\theta_0R_0+d_l\tan\theta\frac{\partial u}{\partial x_2}\right)\right]-d_l\frac{\partial^2 u}{\partial x_2^2}=0$$

After simplifications, we get:

$$-d_l^2\frac{\partial^2 u}{\partial x_2^2}+R_{0c}d_l\tan\theta\frac{\partial u}{\partial x_2}+\theta_0R_{0c}^2=0 \tag{15}$$

Equation (15) has for solution on $\left[L_0-e;L_0-\frac{e}{2}\right]$

$$u(x_2)=A_1+B_1exp\left(\frac{R_{0c}\tan\theta}{d_l}x_2\right)-\frac{\theta_0R_{0c}}{d_l\tan\theta}x_2 \tag{16}$$

where A_1 and B_1 are real constants.

- If $x_2 \in \left[L_0 - \frac{e}{2}; L_0\right]$ then $\frac{\partial \xi}{\partial x_2} = -\tan\theta$ and by the same calculations as in the previous case, we have:

$$u(x_2) = A_2 + B_2 exp\left(-\frac{R_{0c}\tan\theta}{d_l} x_2\right) + \frac{\theta_0 R_{0c}}{d_l \tan\theta} x_2 \tag{17}$$

where A_2 and B_2 are real constants. To summarize, note $Y_c = \left[0; L_0 - e\right]$, $Y_{l1} = \left[L_0 - e; L_0 - \frac{e}{2}\right]$, $Y_{l2} = \left[L_0 - \frac{e}{2}; L_0\right]$, then we have:

Theorem 1.6.
At the front Γ_f, the solution u of P_u^{SH} is of the form:

$$u(x_2) = \begin{cases} u_1(x_2) = \frac{1}{2}\theta_0 \frac{R_{0c}^2}{d_c^2} x_2^2 + C_1 x_2 + C_2 & x_2 \in Y_c \\ u_2(x_2) = A_1 + B_1 exp\left(\frac{R_{0c}\tan\theta}{d_l} x_2\right) - \frac{\theta_0 R_{0c}}{d_l \tan\theta} x_2 & x_2 \in Y_{l1} \\ u_3(x_2) = A_2 + B_2 exp\left(-\frac{R_{0c}\tan\theta}{d_l} x_2\right) + \frac{\theta_0 R_{0c}}{d_l \tan\theta} x_2 & x_2 \in Y_{l2} \end{cases} \tag{18}$$

Determination of the constants

To determine the different constants, we use the following properties: periodicity on Γ_1 and Γ_2, continuity in $L_0 - e$, symmetry in $L_0 - \frac{e}{2}, \cdots$, etc.

- The periodicity condition $u_1(0) = u_3(L_0)$ implies

$$C_2 = A_2 + B_2 exp\left(-\frac{R_{0c}\tan\theta L_0}{d_l}\right) + \frac{\theta_0 R_{0c} L_0}{d_l \tan\theta}$$

- The continuity condition in $L_0 - e$: $u_1(L_0 - e) = u_2(L_0 - e)$ leads to

$$\frac{1}{2}\theta_0 \frac{R_{0c}^2}{d_c^2}(L_0 - e)^2 + C_1(L_0 - e) + C_2 = A_1 + B_1 exp\left(\frac{R_{0c}\tan\theta}{d_l}(L_0 - e)\right) - \frac{\theta_0 R_{0c}}{d_l \tan\theta}(L_0 - e)$$

- From the continuity condition in $L_0 - \frac{e}{2} \equiv b$, we have $u_2(b) = u_3(b)$ and the relation below:

$$A_1 + B_1 exp\left(\frac{R_{0c}\tan\theta}{d_l} b\right) - \frac{\theta_0 R_{0c}}{d_l \tan\theta} b = A_2 + B_2 exp\left(-\frac{R_{0c}\tan\theta}{d_l} b\right) + \frac{\theta_0 R_{0c}}{d_l \tan\theta} b$$

- The symmetry condition on $\left[L_0 - e; L_0\right]$ with respect to the axis $x_2 = L_0 - \frac{e}{2} \equiv b$ implies: $\forall x_2$ such thath $x_2 - b$ and $x_2 + b \in \left[L_0 - e; L_0\right]$, we have $u_2(b - x_2) = u_3(b + x_2)$. That is

$$A_1 + B_1 exp\left(\frac{R_{0c}\tan\theta}{d_l}(b - x_2)\right) - \frac{\theta_0 R_{0c}}{d_l \tan\theta}(b - x_2)$$

$$= A_2 + B_2 exp\left(-\frac{R_{0c}\tan\theta}{d_l}(b + x_2)\right) + \frac{\theta_0 R_{0c}}{d_l \tan\theta}(b + x_2)$$

We then get:

$$A_1 + B_1 exp\left(\frac{R_{0c}\tan\theta}{d_l} b\right) exp\left(-\frac{R_{0c}\tan\theta}{d_l} x_2\right) - \frac{\theta_0 R_{0c} b}{d_l \tan\theta} + \frac{\theta_0 R_{0c}}{d \tan\theta} x_2$$

$$= A_2 + B_2 exp\left(-\frac{R_{0c}\tan\theta}{d_l} b\right) exp\left(-\frac{R_{0c}\tan\theta}{d_l} x_2\right) + \frac{\theta_0 R_{0c} b}{d_l \tan\theta} + \frac{\theta_0 R_{0c}}{d_l \tan\theta} x_2$$

By identification we have:

$$\begin{cases} B_1 exp\left(\frac{R_{0c}\tan\theta}{d_l} b\right) = B_2 exp\left(-\frac{R_{0c}\tan\theta}{d_l} b\right) \\ A_1 - \frac{\theta_0 R_{0c} b}{d_l \tan\theta} = A_2 + \frac{\theta_0 R_{0c} b}{d_l \tan\theta} \end{cases}$$

So we can choose the constants as follows:

$$\begin{cases} A_1 = \dfrac{\theta_0 R_{0c} b}{d_l \tan\theta} \\ \\ B_1 = exp\left(-\dfrac{R_{0c}\tan\theta}{d_l} b\right) \end{cases} \qquad \begin{cases} A_2 = -\dfrac{\theta_0 R_{0c} b}{d_l \tan\theta} \\ \\ B_2 = exp\left(\dfrac{R_{0c}\tan\theta}{d_l} b\right) \end{cases}$$

and we deduce $C_2 = A_2 + B_2 exp\left(-\dfrac{R_{0c}\tan\theta}{d_l} L_0\right) + \dfrac{\theta_0 R_{0c}}{d \tan\theta} L_0$ and

$$C_1 = \frac{1}{(L_0 - e)}\left[A_1 + B_1 exp\left(\frac{R_{0c}\tan\theta}{d_l}(L_0 - e)\right) - \frac{\theta_0 R_{0c}}{d_l \tan\theta}(L_0 - e) - C_2 - \frac{1}{2}\theta_0 \frac{R_{0c}}{d_c^2}(L_0 - e)^2\right]$$

The graphic representation of the function (18) below confirms the change in profile observed and is analogous to that of the Figure 5.

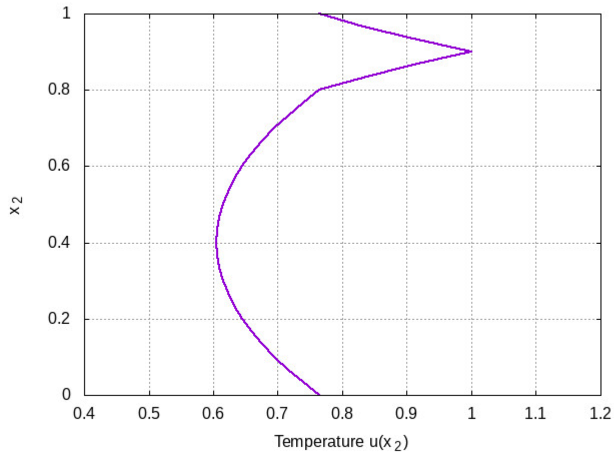

Fig. 7. Analytic temperature profile at the front.

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