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The non-negative P_0 - matrix completion problem for 5×5 matrices specifying digraphs with 5 vertices and 4 arcs for acyclic digraphs

Research Article

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Abstract: The non-negative P_0-matrix completion is considered for 5×5 matrices specifying digraphs with p=5 and q=4.The research determines which of the digraphs with p=5 and q=4 and specifying 5×5 partial matrices have non-negative P_0-completion. Considering the 5×5 matrices with q=4, all the sixty one (61) non-isomorphic digraphs shall be constructed. All the partial non-negative P_0-matrices will be extracted from each digraph. To establish if the pattern has non-negative P_0-completion or not, zero completion will be performed on each of the partial matrix extracted. The study establishes that all acyclic digraphs of an 5×5 matrix with q=4 have non-negative P_0-completion. The matrix completion problem is to find the values of an n x m matrix M, given a sparse and incomplete set of observations. Possible areas of applications include Seismic data reconstruction to recover missing traces when data is sparse and incomplete ,say due to malfunctioned measuring instruments, biased or corrupted traces, ground barriers, or due to financial limitation to access complete data. Others include incomplete market surveys (eg movie ratings to complete missing data so as to recommend appropriately to viewers), weather forecasting from historical data recordings as well as future predictions from computer simulations, reconstruction of images in computer; and finding the positions of sensors in Global Positioning from distances available in a local network.

MSC: 35LXX • 76LXX

Keywords: Graph • Subgraphs • Directed digraph • Cyclic digraph • acyclic digraph • Complete digraph • Path • Cycle • zero completion • Isomorphic digraphs • Partial matrix • Sub-matrix • Principal minor • P_0-matrix • non-negative P_o-matrix

1. Introduction

Adil Akhtar et al [1] studied Graph Theoretic approach to analyze amino acids network. They established that the aliphatic amino acids form a clique. All 3×3 matrices have non-negative completion Hogben [2]. A pattern for 4×4 matrices that includes all diagonal positions has non-negative completion if and only if its digraph is complete when it has a 4-cycle Choi et al [4]. They also showed that any positionally symmetric pattern that includes all diagonal positions and whose graph is an n-cycle has non-negative P_0completion if and only if n≠ 4.

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Azhar A.S angoor[3] et al studied Anti fuzzy graphs. They established that if G Anti fuzzy graph, then $G \to V$ satisfied property Anti fuzzy graph. If G Anti fuzzy graph then $C + \star e$ satisfied property Anti fuzzy graph. All digraphs for p= 5, q= 3 specifying 5×5 partial matrices which are either cycles or acyclic have non-negative P_0 completion Munyiri et al [5].

It is possible to work out the number of digraphs (with points and q lines). Harary et al [6], developed a technique for working out the number of digraphs (with points and q lines) and established the following results. Table 1 shows the number of digraphs with n=5 points and q lines.

Table 1. shows the number of digraphs with n=5 points and q lines.

No. of edges (q)	0	1	2	3	4	5	6	7	8	9	10	Greater or equal to 11
No. of digraphs	1	1	5	16	61	154	379	707	1155	1490	1670	3969

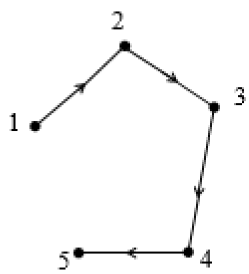
Therefore, there are sixty one (61) non-isomorphic digraphs with 5 points and 4 arcs. Out of this, 56 are acyclic while 5 are cyclic.

All the 56 acyclic non-isomorphic digraphs of 5×5 matrices with p= 5, q =4 are discussed. All the digraphs are assumed to include all diagonal positions. Each digraph shall be considered as a separate case.

Mutti-Ur Rehman[7] studied structured singular values for Bernoulli matrices. It was established that for a smooth matrix family $A : R \to C$ and let $\lambda(t)$ be an eigenvalue of $A(t)$ converges to a simple eigenvalue $\lambda_0 = \lambda(0)$ of $A_0 = A(0)$ as $t \to 0$. Then $\lambda(t)$ is analytic near t=0.
Seyyed et al[8] established that for a particular $n \times n$ matrix $F = [F_{ki,j}]_{i,j=1}^{n}$ then $det(F) = \prod F_{i-1}$.

2. Mathematical Model

Consider the acyclic digraph given below.



The partial non-negative P_0 matrix arising from the digraph is then extracted from the digraph as below:

$$\begin{bmatrix} d_{11} & a_{12} & x_{13} & x_{14} & x_{15} \\ x_{21} & d_{22} & a_{23} & x_{24} & x_{25} \\ x_{31} & x_{32} & d_{33} & a_{34} & a_{45} \\ x_{41} & x_{42} & x_{43} & d_{44} & a_{45} \\ x_{51} & x_{52} & x_{53} & x_{54} & d_{55} \end{bmatrix}$$

The other digraphs and their corresponding partial non-negative P_0 - matrices can be constructed and extracted, respectively, in a similar manner.
By definition of partial non-negative P_0 matrix $d_{11} \geq 0, d_{22} \geq 0, d_{33} \geq 0, d_{44} \geq 0, d_{55} \geq 0$. Next, we compute the principal minors.
DetB(1,2) = $d_{11}d_{22} - a_{12}x_{21}$
DetB(1,3) = $d_{11}d_{13} - x_{13}x_{31}$
DetB(1,4) = $d_{11}d_{44} - x_{14}x_{41}$
DetB(1,5) = $d_{11}d_{55} - x_{15}x_{51}$
DetB(2,3) = $d_{22}d_{33} - a_{23}x_{32}$
DetB(2,4) = $d_{22}d_{44} - x_{24}x_{42}$

DetB(2,5) = $d_{22}d_{55} - x_{25}x_{52}$
DetB(3,4) = $d_{33}d_{44} - a_{34}x_{43}$
DetB(3,5) = $d_{33}d_{55} - x_{35}x_{53}$
DetB(4,5) = $d_{44}d_{55} - a_{45}x_{54}$

$$DetB(1,2,3) = d_{11}d_{22}d_{33} + x_{13}x_{21}x_{32} + a_{12}x_{23}x_{31} - x_{13}d_{22}x_{31} - d_{11}a_{23}x_{32} - a_{12}x_{21}d_{33}$$

$$Det(1,2,4) = d_{11}d_{22}d_{44} + x_{14}x_{21}x_{42} + a_{12}x_{24}x_{41} - x_{14}d_{22}x_{41} - d_{11}x_{24}x_{42} - a_{12}x_{21}d_{44}$$

$$Det(1,2,5) = d_{11}d_{22}d_{55} + x_{15}x_{21}x_{52} + a_{12}x_{25}x_{51} - x_{15}d_{22}x_{51} - d_{11}x_{25}x_{52} - a_{12}x_{21}d_{55}$$

$$DetB(1,3,4) = d_{11}d_{33}d_{44} + x_{14}x_{31}x_{43} + x_{13}x_{34}x_{41} - x_{14}d_{33}x_{41} - d_{11}a_{34}x_{43} - x_{13}x_{31}d_{44}$$

$$DetB(1,3,5) = d_{11}d_{33}d_{55} + x_{15}x_{31}x_{53} + x_{13}x_{35}x_{51} - x_{15}d_{33}x_{51} - d_{11}x_{35}x_{53} - x_{13}x_{31}d_{55}$$

$$DetB(1,4,5) = d_{11}d_{44}d_{55} + x_{15}x_{41}x_{54} + x_{14}a_{45}x_{51} - x_{15}d_{44}x_{51} - d_{11}a_{45}x_{54} - x_{14}x_{41}d_{55}$$

$$DetB(2,3,4) = d_{22}d_{33}d_{44} + x_{24}x_{32}x_{43} + a_{23}a_{34}x_{42} - x_{24}d_{33}x_{42} - d_{22}a_{34}x_{43} - a_{23}x_{32}d_{44}$$

$$DetB(2,3,5) = d_{22}d_{33}d_{55} + x_{25}x_{32}x_{53} + a_{23}x_{35}x_{52} - x_{25}d_{33}x_{52} - d_{22}x_{35}x_{53} - a_{23}x_{32}d_{55}$$

$$DetB(2,4,5) = d_{22}d_{44}d_{55} + x_{25}x_{42}x_{54} + x_{24}a_{45}x_{52} - x_{25}d_{44}x_{52} - d_{22}a_{45}x_{54} - x_{24}x_{42}d_{55}$$

$$DetB(3,4,5) = d_{33}d_{44}d_{55} + x_{35}x_{43}x_{54} + a_{34}a_{45}x_{53} - x_{35}d_{44}x_{53} - d_{33}a_{45}x_{54} - a_{34}x_{43}d_{55}$$

$$\begin{aligned}DetB(1,2,3,4) = d_{11}\,&[d_{22}d_{33}d_{44} + x_{24}x_{32}x_{43} + a_{23}x_{34}x_{42} - x_{24}d_{33}x_{42} - d_{22}x_{34}x_{43} - a_{23}x_{32}\ d_{44}]\\
-a_{12}\,&[x_{21}d_{33}d_{44} + x_{24}x_{31}x_{43} + a_{23}x_{34}x_{41} - x_{24}d_{33}x_{41} - x_{21}a_{34}x_{43} - a_{23}x_{31}d_{44}]\\
+x_{13}\,&[x_{21}x_{32}d_{44} + x_{24}x_{31}x_{42} + d_{22}a_{34}x_{41} - x_{24}x_{32}x_{41} - x_{21}a_{34}x_{42} - d_{22}x_{31}d_{44}]\\
-x_{14}\,&[x_{21}x_{32}x_{43} + x_{23}x_{31}x_{42} + d_{22}d_{33}x_{41} - a_{23}x_{32}x_{41} - x_{21}d_{33}x_{42} - d_{22}x_{31}x_{43}]\end{aligned}$$

$$\begin{aligned}DetB(1,2,3,5) = d_{11}\,&[d_{22}d_{33}d_{55} + x_{25}x_{32}x_{53} + a_{23}x_{35}x_{52} - x_{25}d_{33}x_{52} - d_{22}x_{35}x_{53} - a_{23}x_{32}\ d_{55}]\\
-a_{12}\,&[x_{21}d_{33}d_{55} + x_{25}x_{31}x_{53} + a_{23}x_{35}x_{51} - x_{25}d_{33}x_{41} - x_{21}x_{35}x_{53} - a_{23}x_{31}d_{55}]\\
+x_{13}\,&[x_{21}x_{32}d_{55} + x_{25}x_{31}x_{52} + d_{22}x_{35}x_{51} - x_{25}x_{32}x_{51} - x_{21}x_{35}x_{52} - d_{22}x_{31}d_{55}]\\
-x_{15}\,&[x_{21}x_{32}x_{53} + a_{23}x_{31}x_{52} + d_{22}d_{33}x_{51} - a_{23}x_{32}x_{51} - x_{21}d_{33}x_{52} - d_{22}x_{31}x_{53}]\end{aligned}$$

$$\begin{aligned}DetB(1,2,4,5) = d_{11}\,&[d_{22}d_{44}d_{55} + x_{25}x_{42}x_{54} + x_{24}a_{45}x_{52} - x_{25}d_{44}x_{52} - d_{22}a_{45}x_{54} - x_{24}x_{42}\ d_{55}]\\
-a_{12}\,&[x_{21}d_{44}d_{55} + x_{25}x_{41}x_{54} + x_{24}x_{45}x_{51} - x_{25}d_{44}x_{51} - x_{21}a_{45}x_{54} - x_{24}x_{41}d_{55}]\\
+x_{14}\,&[x_{21}x_{42}d_{55} + x_{25}x_{41}x_{52} + d_{22}a_{45}x_{51} - x_{25}x_{42}x_{51} - x_{21}a_{45}x_{52} - d_{22}x_{41}d_{55}]\\
-x_{15}\,&[x_{21}x_{42}x_{54} + x_{24}x_{41}x_{52} + d_{22}d_{44}x_{51} - x_{24}x_{42}x_{51} - x_{21}d_{44}x_{52} - d_{22}x_{41}x_{54}]\end{aligned}$$

$$\begin{aligned}DetB(1,3,4,5) = d_{11}\,&[d_{33}d_{44}d_{55} + x_{35}x_{43}x_{54} + a_{34}x_{45}x_{53} - x_{35}d_{44}x_{53} - d_{33}a_{45}x_{54} - a_{34}x_{43}\ d_{55}]\\
-x_{13}\,&[x_{31}d_{44}d_{55} + x_{35}x_{41}x_{54} + a_{34}x_{45}x_{51} - x_{35}d_{44}x_{51} - x_{31}a_{45}x_{54} - a_{34}x_{41}d_{55}]\\
+x_{14}\,&[x_{31}x_{43}d_{55} + x_{35}x_{41}x_{53} + d_{33}a_{45}x_{51} - x_{35}x_{43}x_{51} - x_{31}a_{45}x_{53} - d_{33}x_{41}d_{55}]\\
-x_{15}\,&[x_{31}x_{43}x_{54} + a_{34}x_{41}x_{53} + d_{33}d_{44}x_{51} - a_{34}x_{43}x_{51} - x_{31}d_{44}x_{53} - d_{33}x_{41}x_{54}]\end{aligned}$$

$$\begin{aligned}DetB(2,3,4,5) = d_{22}\,&[d_{33}d_{44}d_{55} + x_{35}x_{43}x_{54} + a_{34}a_{45}x_{53} - x_{35}d_{44}x_{53} - d_{33}a_{45}x_{54} - a_{34}x_{43}\ d_{55}]\\
-a_{23}\,&[x_{32}d_{44}d_{55} + x_{35}x_{42}x_{54} + a_{34}x_{45}x_{52} - x_{35}d_{44}x_{52} - x_{32}a_{45}x_{54} - a_{34}x_{42}d_{55}]\\
+x_{24}\,&[x_{32}x_{43}d_{55} + x_{35}x_{42}x_{53} + d_{33}a_{45}x_{52} - x_{35}x_{43}x_{52} - x_{32}a_{45}x_{53} - d_{33}x_{42}d_{55}]\\
-x_{25}\,&[x_{32}x_{43}x_{54} + a_{34}x_{42}x_{53} + d_{33}d_{44}x_{52} - a_{34}x_{43}x_{52} - x_{32}d_{44}x_{53} - d_{33}x_{42}x_{54}]\end{aligned}$$

$$\begin{aligned}DetB(1,2,3,4,5) = d_{11}\,(d_{22}\,&[d_{33}d_{44}d_{55} + x_{35}x_{43}x_{54} + a_{34}a_{45}x_{53} - x_{35}d_{44}x_{53} - d_{33}a_{45}x_{54} - a_{34}x_{43}d_{55}]\\
-a_{23}\,&[x_{32}d_{44}d_{55} + x_{35}x_{42}x_{54} + a_{34}a_{45}x_{52} - x_{35}d_{44}x_{52} - x_{32}a_{45}x_{54} - a_{34}x_{42}d_{55}]\\
+x_{24}\,&[x_{32}x_{43}d_{55} + x_{35}x_{42}x_{53} + d_{33}a_{45}x_{52} - x_{35}x_{43}x_{52} - x_{32}a_{45}x_{53} - d_{33}x_{42}d_{55}]\\
-x_{25}\,&[x_{32}x_{43}x_{54} + a_{34}x_{42}x_{53} + d_{33}d_{44}x_{52} - a_{34}x_{43}x_{52} - x_{32}d_{44}x_{53} - d_{33}x_{42}x_{54}])\end{aligned}$$

$$\begin{aligned}-a_{12}\,(x_{21}\,&[d_{33}d_{44}d_{55} + x_{35}x_{43}x_{54} + a_{34}a_{45}x_{53} - x_{35}d_{44}x_{53} - d_{33}a_{45}x_{54} - a_{34}x_{43}d_{55}]\\
-a_{23}\,&[x_{31}d_{44}d_{55} + x_{35}x_{41}x_{54} + a_{34}a_{45}x_{51} - x_{35}d_{44}x_{51} - x_{31}a_{45}x_{54} - a_{34}x_{41}d_{55}]\\
+x_{24}\,&[x_{31}x_{43}d_{55} + x_{35}x_{41}x_{53} + d_{33}a_{45}x_{51} - x_{35}x_{43}x_{51} - x_{31}a_{45}x_{53} - d_{33}x_{41}d_{55}]\\
-x_{25}\,&[x_{31}x_{43}x_{54} + a_{34}x_{41}x_{53} + d_{33}d_{44}x_{51} - a_{34}x_{43}x_{51} - x_{31}d_{44}x_{53} - d_{33}x_{41}x_{54}])\end{aligned}$$

$$\begin{aligned}+x_{13}\,(x_{21}\,&[x_{32}d_{44}d_{55} + x_{35}x_{41}x_{54} + a_{34}a_{45}x_{52} - x_{35}d_{44}x_{52} - x_{32}a_{45}x_{54} - a_{34}x_{42}d_{55}]\\
-d_{22}\,&[x_{31}d_{44}d_{55} + x_{35}x_{41}x_{54} + a_{34}a_{45}x_{51} - x_{35}d_{44}x_{51} - x_{31}a_{45}x_{54} - a_{34}x_{41}d_{55}]\\
+x_{24}\,&[x_{31}x_{41}d_{55} + x_{35}x_{41}x_{52} + x_{32}a_{45}x_{51} - x_{35}x_{42}x_{51} - x_{31}a_{45}x_{52} - x_{32}x_{41}d_{55}]\\
-x_{25}\,&[x_{31}x_{42}x_{54} + a_{34}x_{41}x_{52} + x_{32}d_{44}x_{51} - a_{34}x_{41}x_{51} - x_{31}d_{44}x_{52} - x_{32}x_{41}x_{54}])\end{aligned}$$

$$+x_{14}\,(x_{21}\,[x_{32}x_{43}d_{55}+x_{35}x_{42}x_{53}+d_{33}a_{45}x_{52}-x_{35}x_{53}x_{52}-x_{32}a_{45}x_{53}-\ d_{33}x_{42}d_{55}]$$
$$-d_{22}\,[x_{31}x_{43}d_{55}+x_{35}x_{41}x_{53}+d_{33}a_{45}x_{51}-x_{35}x_{43}x_{51}-x_{31}a_{45}x_{53}-\ d_{33}x_{41}d_{55}]$$
$$+a_{23}\,[x_{31}x_{42}d_{55}+x_{35}x_{41}x_{52}+x_{32}a_{45}x_{51}-x_{35}x_{42}x_{51}-x_{31}a_{45}x_{52}-\ x_{41}x_{32}d_{55}]$$
$$-x_{25}\,[x_{31}x_{42}x_{53}+d_{33}x_{41}x_{52}+x_{32}x_{43}x_{51}-d_{33}x_{42}x_{51}-x_{31}x_{43}x_{52}-\ x_{32}x_{41}x_{53}])$$

$$+x_{15}\,(x_{21}\,[x_{32}x_{43}x_{54}+a_{34}x_{42}x_{53}+d_{33}d_{44}x_{52}-a_{34}x_{43}x_{52}-x_{32}d_{44}x_{53}-\ d_{33}x_{42}x_{54}]$$
$$-d_{22}\,[x_{31}x_{43}x_{54}+a_{34}x_{41}x_{53}+d_{33}d_{44}x_{51}-a_{34}x_{43}x_{51}-x_{31}d_{44}x_{53}-\ d_{33}x_{41}x_{54}]$$
$$+a_{23}\,[x_{31}x_{42}x_{54}+a_{34}x_{41}x_{52}+x_{32}d_{44}x_{51}-a_{34}x_{42}x_{51}-x_{31}d_{44}x_{52}-\ x_{32}x_{41}x_{54}]$$
$$-x_{24}\,[x_{31}x_{42}x_{53}+d_{33}x_{41}x_{52}+x_{32}x_{43}x_{51}-d_{33}x_{42}x_{51}-x_{31}x_{43}x_{52}-\ x_{32}x_{41}x_{53}])$$

Setting the unspecified entries to zero, that is

$$x_{13}=x_{14}=x_{15}=x_{21}=x_{24}=x_{25}=x_{31}=x_{32}=x_{35}=x_{41}=x_{42}=x_{43}=$$

$$x_{51}=x_{52}=x_{53}=x_{54}=0,$$

we have

$$DetB(1,2)=d_{11}d_{22}\geq 0$$

$$DetB(1,3)=d_{11}d_{33}\geq 0$$

$$DetB(1,4)=d_{11}d_{44}\geq 0$$

$$DetB(1,5)=d_{11}d_{55}\geq 0$$

$$DetB(2,3)=d_{22}d_{33}\geq 0$$

$$DetB(2,4)=d_{22}d_{44}\geq 0$$

$$DetB(2,5)=d_{22}d_{55}\geq 0$$

$$DetB(3,4)=d_{34}d_{44}\geq 0$$

$$DetB(3,5)=d_{33}d_{55}\geq 0$$

$$DetB(4,5)=d_{44}d_{55}\geq 0$$

$$DetB(1,2,3)=d_{11}d_{22}d_{33}\geq 0$$

$$DetB(1,2,4)=d_{11}d_{22}d_{44}\geq 0$$

$$DetB(1,2,5)=d_{11}d_{22}d_{55}\geq 0$$

$$DetB(1,3,4)=d_{11}d_{33}d_{55}\geq 0$$

$$DetB(1,3,5)=d_{11}d_{33}d_{55}\geq 0$$

$$DetB(1,4,5)=d_{11}d_{44}d_{55}\geq 0$$

$$DetB(2,3,4)=d_{22}d_{33}d_{44}\geq 0$$

$$DetB(2,3,5)=d_{22}d_{33}d_{55}\geq 0$$

$$DetB(2,4,5)=d_{22}d_{44}d_{55}\geq 0$$

$$DetB(3,4,5)=d_{22}d_{33}d_{45}\geq 0$$

$$DetB(1,2,3,4)=d_{11}d_{22}d_{33}d_{44}\geq 0$$

$$DetB(1,2,3,5)=d_{11}d_{22}d_{33}d_{55}\geq 0$$

$$DetB(1,2,4,5)=d_{11}d_{22}d_{44}d_{55}\geq 0$$

$$DetB(1,3,4,5)=d_{11}d_{33}d_{44}d_{55}\geq 0$$

$$DetB(2,3,4,5)=d_{22}d_{33}d_{44}d_{55}\geq 0$$

$$Det\,B(1,2,3,4,5)=d_{11}d_{22}d_{33}d_{44}d_{55}\geq 0$$

Hence, the determinants are non-negative and so there is a zero completion into a non-negative P_0-matrix for the digraph.

4 Results

Hence all principal minors are non-negative and therefore the partial matrix has zero completion into non-negative P_0- matrix.

4 Conclusions

A similar procedure is carried out on all the other remaining acyclic digraphs to determine if there is completion into a non-negative -matrix or not. It was established that all acyclic digraphs of a 5×5 matrix with q=4 have non-negative P_0-completion.

4 Validation

When q=4 and n=5 the results agrees with those of Munyiri et al [3] where q=3 and n=5. The results obtained in these research can be applied in filling partially filled data in market surveys to predict markets trends.

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