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Numerical solutions of telegraph and laplace equations on cantor sets using local fractional laplace decomposition method

Research Article

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Abstract: In this paper, the local fractional Laplace decomposition method is implemented to obtain approximate analytical solution of the telegraph and Laplace equations on Cantor sets. This method is a combination of the Yang-Laplace transform and the Adomian decomposition method. Some examples are given to illustrate the efficiency and accuracy of the proposed method to obtain analytical solutions to differential equations within the local fractional derivatives.
is implemented to obtain

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1. Introduction

The theory of local fractional integrals and derivatives was dealing with fractal functions [1–12], and was successfully applied in fractional Fokker-Planck equation [1, 2], anomalous diffusion and relaxation equation in fractal space [3], fractal-time dynamical systems [4, 5], fractal elasticity [6, 7], local fractional diffusion equation, local fractional Laplace equation [13], local fractional ordinary differential equations [11, 12], local fractional partial differential equation [11–13], local fractional integral equations [16, 21], fractional Brownian motion in local fractional derivatives sense [12], fractal signals [11, 12, 14, 17], local fractional Fourier analysis [14], local fractional Fourier series [11, 12], fractal wave equation [13], Yang-Fourier transform [13, 14, 16], Yang-Laplace transform [13, 15], discrete Yang-Fourier transform [17], local fractional Stieltjes transform in fractal space [19], local fractional Mellin transform in fractal space [20], Local fractional Z transform in fractal space [15], local fractional short time transforms [11, 12] and local fractional wavelet transform [11, 12]. In order to deal with local fractional ordinary and partial differential equations, there are some developed technologies, for example, the local fractional variational iteration method [22, 23], the local fractional Fourier series method [24, 25], the Cantor-type cylindrical-coordinate method [26], the Yang-Fourier transform [27], and the Yang-Laplace transform [28], and the local fractional function decomposition method [29, 30].
In this work we use the local fractional function decomposition method which is a combination of the Adomian decomposition method and Yang-Laplace transform [29–33].

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2. Mathematical fundamentals

In this section, we present the basic theory of local fractional calculus and concepts of local fractional Laplace transform (see [11, 12, 21, 22]).

Definition 2.1.
Suppose that there is the relation

$$|f(x)-f(x_0)|<\epsilon^{\alpha}, \quad 0<\alpha\leq 1, \tag{1}$$

with $|x-x_0|<\delta$, for $\varepsilon,\delta>0$ and $\varepsilon,\delta\in R$, then the function $f(x)$ is called local fractional continuous at $x=x_0$ and it is denoted by $\lim\limits_{x\to x_0} f(x)=f(x_0)$.

Definition 2.2.
Suppose that the function $f(x)$ satisfies condition (1), for $x\in(a,b)$; it is so called local fractional continuous on the interval (a,b), denoted by $f(x)\in C_{\alpha}(a,b)$.

Definition 2.3.
In the fractal space, let $f(x)\in C_{\alpha}(a,b)$, local fractional derivative of $f(x)$ of order α at $x=x_0$ is given by

$$f^{(\alpha)}(x_0)=\frac{d^{\alpha}}{dx^{\alpha}}f(x)|_{x=x_0}=\lim_{x\to x_0}\frac{\triangle^{\alpha}(f(x)-f(x_0))}{(x-x_0)^{\alpha}} \tag{2}$$

where $\triangle^{\alpha}(f(x)-f(x_0)\cong\Gamma(\alpha+1)(f(x)-f(x_0))$.

Definition 2.4.
A partition of the interval $[a,b]$ is denoted as (t_j,t_{j+1}), $j=0,...,N-1$, $t_0=a$ and $t_N=b$ with $\triangle t_j=t_{j+1}-t_j$ and $\triangle t=\max\{\triangle t_0,\triangle t_1,\ldots\}$. Local fractional integral of $f(x)$ in the interval $[a,b]$ is given by

$${}_aI_b^{(\alpha)}f(x)=\frac{1}{\Gamma(1+\alpha)}\int_a^b f(t)(dt)^{\alpha}=\frac{1}{\Gamma(1+\alpha)}\lim_{\triangle t\longrightarrow 0}\sum_{j=0}^{N-1}f(t_j)(\triangle t_j)^{\alpha}. \tag{3}$$

Definition 2.5.
Let $\frac{1}{\Gamma(1+\alpha)}\int_0^{\infty}|f(x)|(dx)^{\alpha}<k<\infty$. The Yang-Laplace transform of $f(x)$ is given by

$$L_{\alpha}\{f(x)\}=f_s^{L,\alpha}(s)=\frac{1}{\Gamma(1+\alpha)}\int_0^{\infty}E_{\alpha}(-s^{\alpha}x^{\alpha})f(x)(dx)^{\alpha}, \quad 0<\alpha\leq 1, \tag{4}$$

where the latter integral converges and $s^{\alpha}\in R^{\alpha}$.

Definition 2.6.
The inverse formula of the Yang-Laplace transform of $f(x)$ is given by

$$L_{\alpha}^{-1}\left(f_s^{L,\alpha}(s)\right)=f(x)=\frac{1}{(2\pi)^{\alpha}}\int_{\beta-i\omega}^{\beta+i\omega}E_{\alpha}(s^{\alpha}x^{\alpha})f_s^{L,\alpha}(s)(ds)^{\alpha}, \quad 0<\alpha\leq 1 \tag{5}$$

where $s^{\alpha}=\beta^{\alpha}+i^{\alpha}\omega^{\alpha}$; fractal imaginary unit i^{α} and $Re(s)=\beta>0$.

The properties for local fractional Laplace transform used in the paper are given as

$$L_{\alpha}\{af(x)+bg(x)\}=af_s^{L,\alpha}(s)+bg_s^{L,\alpha}(s) \tag{6}$$

$$L_{\alpha}\{E_{\alpha}(c^{\alpha}x^{\alpha})f(x)\}=f_s^{L,\alpha}(s-c) \tag{7}$$

$$L_{\alpha}\{f^{(k\alpha)}(x)\}=s^{k\alpha}f_s^{L,\alpha}(s)-s^{(k-1)\alpha}f(0)-s^{(k-2)\alpha}f^{(\alpha)}(0)-...-f^{((k-1)\alpha)}(0) \tag{8}$$

$$L_{\alpha}\{E_{\alpha}(c^{\alpha}x^{\alpha})\}=\frac{1}{s^{\alpha}-c^{\alpha}} \tag{9}$$

$$L_{\alpha}\{\sin_{\alpha}(c^{\alpha}x^{\alpha})\}=\frac{c^{\alpha}}{s^{2\alpha}+c^{2\alpha}} \tag{10}$$

$$L_{\alpha}\{x^{k\alpha}\}=\frac{\Gamma(1+k\alpha)}{s^{(k+1)\alpha}} \tag{11}$$

3. Local fractional Laplace decomposition method

Let us consider the following linear operator with local fractional derivative:

$$Ł_{\alpha} u(x,t)+R_{\alpha} u(x,t)=h(x,t), \tag{12}$$

where $Ł_{\alpha}=\dfrac{\partial^{k\alpha}}{\partial x^{k\alpha}}$ denotes the linear local fractional differential operator, R_{α} is the remaining linear operator, and $h(x,t)$ is a source term. Taking Yang-Laplace transform on Eq. (12), we obtain

$$L_{\alpha}\{Ł_{\alpha} u(x,t)\}+L_{\alpha}\{R_{\alpha} u(x,t)\}=L_{\alpha}\{h(x,t)\}. \tag{13}$$

By applying the local fractional Laplace transform differentiation property, we have

$$s^{k\alpha}L_{\alpha}\{u(x,t)\}-s^{(k-1)\alpha}u(0,t)-\cdots-u^{((k-1)\alpha)}(0,t)+L_{\alpha}\{R_{\alpha}u(x,t)\}=L_{\alpha}\{h(x,t)\}. \tag{14}$$

or

$$L_{\alpha}\{u(x,t)\}=\frac{1}{s^{\alpha}}u(0,t)+\ldots+\frac{1}{s^{k\alpha}}u^{((k-1)\alpha)}(0,t)+\frac{1}{s^{k\alpha}}L_{\alpha}\{h(x,t)\}-\frac{1}{s^{k\alpha}}L_{\alpha}\{R_{\alpha}u(x,t)\}. \tag{15}$$

Taking the inverse of local fractional Laplace transform on Eq. (15), we have

$$\begin{aligned}u(x,t)=u(0,t)+\frac{x^{\alpha}}{\Gamma(1+\alpha)}u^{(\alpha)}(0,t)+\ldots+\frac{x^{(k-1)\alpha}}{\Gamma(1+(k-1)\alpha)}u^{((k-1)\alpha)}(0,t)+L_{\alpha}^{-1}\left(\frac{1}{s^{k\alpha}}L_{\alpha}\{h(x,t)\}\right)\\ -L_{\alpha}^{-1}\left(\frac{1}{s^{k\alpha}}L_{\alpha}\{R_{\alpha}u(x,t)\}\right).\end{aligned} \tag{16}$$

We are going to represent the solution in an infinite series given below:

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t). \tag{17}$$

Substituting Eq. (17) into Eq. (16), which give us this result

$$\begin{aligned}\sum_{n=0}^{\infty}u_n(x,t)=u(0,t)+\frac{x^{\alpha}}{\Gamma(1+\alpha)}u^{(\alpha)}(0,t)+\ldots+\frac{x^{(k-1)\alpha}}{\Gamma(1+(k-1)\alpha)}u^{((k-1)\alpha)}(0,t)+L_{\alpha}^{-1}\left(\frac{1}{s^{k\alpha}}L_{\alpha}\{h(x,t)\}\right)\\ -L_{\alpha}^{-1}\left(\frac{1}{s^{k\alpha}}L_{\alpha}\left\{R_{\alpha}\sum_{n=0}^{\infty}u_n(x,t)\right\}\right).\end{aligned} \tag{18}$$

When we compare the left and right hand sides of Eq. (18) we obtain

$$\begin{aligned}u_0(x,t)&=u(0,t)+\frac{x^{\alpha}}{\Gamma(1+\alpha)}u^{(\alpha)}(0,t)+\ldots+\frac{x^{(k-1)\alpha}}{\Gamma(1+(k-1)\alpha)}u^{((k-1)\alpha)}(0,t)+L_{\alpha}^{-1}\left(\frac{1}{s^{k\alpha}}L_{\alpha}\{h(x,t)\}\right),\\ u_1(x,t)&=-L_{\alpha}^{-1}\left(\frac{1}{s^{k\alpha}}L_{\alpha}\{R_{\alpha}u_0(x,t)\}\right),\\ u_2(x,t)&=-L_{\alpha}^{-1}\left(\frac{1}{s^{k\alpha}}L_{\alpha}\{R_{\alpha}u_1(x,t)\}\right)\end{aligned} \tag{19}$$

The recursive relation, in general form is

$$\begin{aligned}u_0(x,t)&=u(0,t)+\frac{x^{\alpha}}{\Gamma(1+\alpha)}u^{(\alpha)}(0,t)+\ldots+\frac{x^{(k-1)\alpha}}{\Gamma(1+(k-1)\alpha)}u^{((k-1)\alpha)}(0,t)+L_{\alpha}^{-1}\left(\frac{1}{s^{k\alpha}}L_{\alpha}\{h(x,t)\}\right)\\ u_n+1(x,t)&=-L_{\alpha}^{-1}\left(\frac{1}{s^{k\alpha}}L_{\alpha}\{R_{\alpha}u_n(x,t)\}\right),\end{aligned} \tag{20}$$

4. Applications to telegraph and Laplace equations on cantor sets

In this section four examples for Telegraph and Laplace equations on Cantor sets will demonstrate the simplicity and the efficiency of local fractional Laplace decomposition method.

Example 4.1.
Let us consider the telegraph equation on Cantor sets

$$\frac{\partial^{2\alpha} u(x,y)}{\partial x^{2\alpha}} = \frac{\partial^{2\alpha} u(x,y)}{\partial y^{2\alpha}} + \frac{\partial^{\alpha} u(x,y)}{\partial y^{\alpha}} - u(x,y), \tag{21}$$

with the initial condition

$$u(0,y) = E_\alpha(-y^\alpha), \frac{\partial^{\alpha} u(0,y)}{\partial x^{\alpha}} = E_\alpha(-y^\alpha). \tag{22}$$

In view of Eqs. (20) and (21) the local fractional iteration algorithm can be written as follows:

$$\begin{aligned} u_0(x,y) &= E_\alpha(-y^\alpha) + \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(-y^\alpha), \\ u_{n+1}(x,y) &= L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} L_\alpha \left\{ \frac{\partial^{2\alpha} u_n(x,y)}{\partial y^{2\alpha}} + \frac{\partial^{\alpha} u_n(x,y)}{\partial y^{\alpha}} - u_n(x,y) \right\}\right), n \geq 0. \end{aligned} \tag{23}$$

Therefore, from Eq. (23) we give the components as follows:

$$u_0(x,y) = E_\alpha(-y^\alpha) + \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(-y^\alpha), \tag{24}$$

$$\begin{aligned} u_1(x,y) &= L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} L_\alpha \left\{ \frac{\partial^{2\alpha} u_0(x,y)}{\partial y^{2\alpha}} + \frac{\partial^{\alpha} u_0(x,y)}{\partial y^{\alpha}} - u_0(x,y) \right\}\right) \\ &= -\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} E_\alpha(-y^\alpha) - \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} E_\alpha(-y^\alpha), \end{aligned} \tag{25}$$

$$\begin{aligned} u_2(x,y) &= L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} L_\alpha \left\{ \frac{\partial^{2\alpha} u_1(x,y)}{\partial y^{2\alpha}} + \frac{\partial^{\alpha} u_1(x,y)}{\partial y^{\alpha}} - u_1(x,y) \right\}\right) \\ &= \frac{x^{4\alpha}}{\Gamma(1+4\alpha)} E_\alpha(-y^\alpha) + \frac{x^{5\alpha}}{\Gamma(1+5\alpha)} E_\alpha(-y^\alpha), \end{aligned} \tag{26}$$

$$\begin{aligned} u_3(x,y) &= L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} L_\alpha \left\{ \frac{\partial^{2\alpha} u_2(x,y)}{\partial y^{2\alpha}} + \frac{\partial^{\alpha} u_2(x,y)}{\partial y^{\alpha}} - u_2(x,y) \right\}\right) \\ &= -\frac{x^{6\alpha}}{\Gamma(1+6\alpha)} E_\alpha(-y^\alpha) - \frac{x^{7\alpha}}{\Gamma(1+7\alpha)} E_\alpha(-y^\alpha), \end{aligned} \tag{27}$$

$$\vdots$$

Consequently, we obtain

$$\begin{aligned} u(x,y) &= E_\alpha(-y^\alpha)\left(1 - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{x^{4\alpha}}{\Gamma(1+4\alpha)} - \cdots\right) + E_\alpha(-y^\alpha)\left(\frac{x^{\alpha}}{\Gamma(1+\alpha)} - \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} + \cdots\right) \\ &= E_\alpha(-y^\alpha)\cos_\alpha(x^\alpha) + E_\alpha(-y^\alpha)\sin_\alpha(x^\alpha). \end{aligned} \tag{28}$$

Example 4.2.
Consider the telegraph equation on Cantor sets

$$\frac{\partial^{2\alpha} u(x,y)}{\partial x^{2\alpha}} = \frac{\partial^{2\alpha} u(x,y)}{\partial y^{2\alpha}} + 2\frac{\partial^{\alpha} u(x,y)}{\partial y^{\alpha}} + u(x,y), \tag{29}$$

with the initial condition

$$u(0,y) = 1 - E_\alpha(-y^\alpha), \frac{\partial^{\alpha} u(0,y)}{\partial x^{\alpha}} = 0. \tag{30}$$

In view of Eqs. (20) and (29) the local fractional iteration algorithm can be written as follows:

$$\begin{aligned} u_0(x,y) &= 1 - E_\alpha(-y^\alpha), \\ u_{n+1}(x,y) &= L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} L_\alpha \left\{ \frac{\partial^{2\alpha} u_n(x,y)}{\partial y^{2\alpha}} + 2\frac{\partial^{\alpha} u_n(x,y)}{\partial y^{\alpha}} + u_n(x,y) \right\}\right), n \geq 0. \end{aligned} \tag{31}$$

Therefore, from Eq. (31) we give the components as follows:

$$u_0(x,y) = 1 - E_\alpha(-y^\alpha), \tag{32}$$

$$u_1(x,y) = L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} L_\alpha\left\{\frac{\partial^{2\alpha} u_0(x,y)}{\partial y^{2\alpha}} + 2\frac{\partial^\alpha u_0(x,y)}{\partial y^\alpha} + u_0(x,y)\right\}\right)$$

$$= L_\alpha^{-1}\left(\frac{1}{s^{3\alpha}}\right) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}, \tag{33}$$

$$u_2(x,y) = L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} L_\alpha\left\{\frac{\partial^{2\alpha} u_1(x,y)}{\partial y^{2\alpha}} + 2\frac{\partial^\alpha u_1(x,y)}{\partial y^\alpha} + u_1(x,y)\right\}\right)$$

$$= L_\alpha^{-1}\left(\frac{1}{s^{5\alpha}}\right) = \frac{x^{4\alpha}}{\Gamma(1+4\alpha)}, \tag{34}$$

$$u_3(x,y) = L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} L_\alpha\left\{\frac{\partial^{2\alpha} u_2(x,y)}{\partial y^{2\alpha}} + 2\frac{\partial^\alpha u_2(x,y)}{\partial y^\alpha} + u_2(x,y)\right\}\right)$$

$$= L_\alpha^{-1}\left(\frac{1}{s^{7\alpha}}\right) = \frac{x^{6\alpha}}{\Gamma(1+6\alpha)}, \tag{35}$$

$$\vdots$$

Consequently, we obtain

$$u(x,y) = \left(1 + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{x^{4\alpha}}{\Gamma(1+4\alpha)} + \cdots\right) - E_\alpha(-y^\alpha)$$

$$= \cosh_\alpha(x^\alpha) - E_\alpha(-y^\alpha). \tag{36}$$

Example 4.3.

The local fractional Laplace equation is presented as

$$\frac{\partial^{2\alpha} u(x,y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u(x,y)}{\partial y^{2\alpha}} = 0, \tag{37}$$

with the initial condition

$$u(0,y) = -E_\alpha(y^\alpha), \frac{\partial^\alpha u(0,y)}{\partial x^\alpha} = 0. \tag{38}$$

In view of Eqs. (20) and (37) the local fractional iteration algorithm can be written as follows:

$$u_0(x,y) = -E_\alpha(y^\alpha),$$

$$u_{n+1}(x,y) = -L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} L_\alpha\left\{\frac{\partial^{2\alpha} u_n(x,y)}{\partial y^{2\alpha}}\right\}\right), n \geq 0. \tag{39}$$

Therefore, from Eq. (39) we give the components as follows:

$$u_0(x,y) = -E_\alpha(y^\alpha), \tag{40}$$

$$u_1(x,y) = -L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} L_\alpha\left\{\frac{\partial^{2\alpha} u_0(x,y)}{\partial y^{2\alpha}}\right\}\right)$$

$$= L_\alpha^{-1}\left(\frac{E_\alpha(y^\alpha)}{s^{3\alpha}}\right) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} E_\alpha(y^\alpha), \tag{41}$$

$$u_2(x,y) = -L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} L_\alpha\left\{\frac{\partial^{2\alpha} u_1(x,y)}{\partial y^{2\alpha}}\right\}\right)$$

$$= -L_\alpha^{-1}\left(\frac{E_\alpha(y^\alpha)}{s^{5\alpha}}\right) = -\frac{x^{4\alpha}}{\Gamma(1+4\alpha)} E_\alpha(y^\alpha), \tag{42}$$

$$u_3(x,y) = -L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} L_\alpha\left\{\frac{\partial^{2\alpha} u_2(x,y)}{\partial y^{2\alpha}}\right\}\right)$$

$$= L_\alpha^{-1}\left(\frac{E_\alpha(y^\alpha)}{s^{7\alpha}}\right) = \frac{x^{6\alpha}}{\Gamma(1+6\alpha)} E_\alpha(y^\alpha), \tag{43}$$

$$\vdots$$

Consequently, we obtain

$$\begin{aligned} u(x,y) &= -E_\alpha(y^\alpha)\left(1-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{x^{4\alpha}}{\Gamma(1+4\alpha)}+\cdots\right) \\ &= -E_\alpha(y^\alpha)\cos_\alpha(x^\alpha). \end{aligned} \tag{44}$$

Example 4.4.

We consider the following local fractional Laplace equation

$$\frac{\partial^{2\alpha}u(x,y)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}u(x,y)}{\partial y^{2\alpha}}=0, \tag{45}$$

with the initial condition

$$u(0,y)=0, \frac{\partial^\alpha u(0,y)}{\partial x^\alpha}=-E_\alpha(y^\alpha). \tag{46}$$

In view of Eqs. (20) and (45) the local fractional iteration algorithm can be written as follows:

$$\begin{aligned} u_0(x,y) &= -\frac{x^\alpha}{\Gamma(1+\alpha)}E_\alpha(y^\alpha), \\ u_{n+1}(x,y) &= -L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}L_\alpha\left\{\frac{\partial^{2\alpha}u_n(x,y)}{\partial y^{2\alpha}}\right\}\right), n\geq 0. \end{aligned} \tag{47}$$

Therefore, from Eq. (47) we give the components as follows:

$$u_0(x,y) = -\frac{x^\alpha}{\Gamma(1+\alpha)}E_\alpha(y^\alpha), \tag{48}$$

$$\begin{aligned} u_1(x,y) &= -L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}L_\alpha\left\{\frac{\partial^{2\alpha}u_0(x,y)}{\partial y^{2\alpha}}\right\}\right) \\ &= L_\alpha^{-1}\left(\frac{E_\alpha(y^\alpha)}{s^{4\alpha}}\right)=\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}E_\alpha(y^\alpha), \end{aligned} \tag{49}$$

$$\begin{aligned} u_2(x,y) &= -L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}L_\alpha\left\{\frac{\partial^{2\alpha}u_1(x,y)}{\partial y^{2\alpha}}\right\}\right) \\ &= -L_\alpha^{-1}\left(\frac{E_\alpha(y^\alpha)}{s^{6\alpha}}\right)=-\frac{x^{5\alpha}}{\Gamma(1+5\alpha)}E_\alpha(y^\alpha), \end{aligned} \tag{50}$$

$$\begin{aligned} u_3(x,y) &= -L_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}L_\alpha\left\{\frac{\partial^{2\alpha}u_2(x,y)}{\partial y^{2\alpha}}\right\}\right) \\ &= L_\alpha^{-1}\left(\frac{E_\alpha(y^\alpha)}{s^{8\alpha}}\right)=\frac{x^{7\alpha}}{\Gamma(1+7\alpha)}E_\alpha(y^\alpha), \end{aligned} \tag{51}$$

$$\vdots$$

Consequently, we obtain

$$\begin{aligned} u(x,y) &= -E_\alpha(y^\alpha)\left(\frac{x^{\alpha}}{\Gamma(1+\alpha)}-\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}+\cdots\right) \\ &= -E_\alpha(y^\alpha)\sin_\alpha(x^\alpha). \end{aligned} \tag{52}$$

5. Conclusions

In this work, the local fractional Laplace decomposition method is demonstrated as an effective method for solutions of a wide class of problems. Analytical solutions of the telegraph and Laplace equations on Cantor sets involving local fractional derivatives are successfully developed by recurrence relations resulting in convergent series solutions.

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