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# Effect of inclined parallel Riga plates and dusty particles on unsteady MHD Stokes flow of a dusty fluid

**Research Article**

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**Abstract:** In this study, the effect of inclined parallel Riga plates and dusty particles on unsteady magnetohydrodynamic stokes flow of a dusty fluid is investigated. Equations were derived from Maxwell's equation, Navier-Strokes equation, continuity equation, and energy equation then solved through Explicit finite difference method after similarity analysis. Graphical and tabular solutions of velocity are presented. It is established that as the angle of inclination increases, the velocity of fluid and dust particles increases. Increasing the concentration of dusty particles lowers the velocity.

**MSC:** 76W05 • 65M06

**Keywords:** MHD • Dusty fluid • inclined angle • Riga plate



## 1. Introduction

A fluid is a substance perceived to be without fixed shape and which yields easily to external pressure. It can therefore deform under the action of shear force. MHD is derived from existence of a magnetic field and an electrically conducting liquid undergoing movement. The induced magnetic field appear to disturb both the original magnetic field and motion of an induced electric field. This leads to electromagnetic force being produced in the field flowing across a transverse magnetic field and then magnetic field combines to produce a force that resists the motion of the fluid [11].

The interactions of the fluid movement and magnetic forces can be deduced mathematically through solutions of continuity equations, combination of Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism [9].

Rajeev Kumar et al [14] studied Fluid flow through porous medium in a Horizontal channel in inclined Magnetic Field. It was established through the graphed work that magnetic field slows down the fluid flow profile. Sasikala et al [12] researched on stokes flow of dusty fluid. The MHD was unsteady and flow was between two parallel plates in presence of magnetic field. They looked at physical parameters that affect velocity such as density, pressure, time and dust particles. Their findings indicated that the velocity of dust particles exceeded that of fluid particles. Additionally, they noticed that a rise in density and the number of dust particles led to a decrease in fluid velocity. Low et al [10] established that increasing volume fraction of both nano particle and volume in dust particle enhances the velocity

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profiles of the flow. Gireesha et al [5] observed that flow of nanofluid particles were parallel to that of dust. The velocity profiles of both fluid and dust particles, were noted to move with higher momentum as they near the axis of the flow.

Delhi Babu et al,in [3] sort to find the impact of velocity potential in a two dimensional steady MHD stokes flow. The plates were taken to be parallel and the flow was under angular velocity with one plate mobile but uniform while the separate plate at rest.There was uniform suction at the resting plate. Angular velocity was a point of focus in [1] when investigating steady stokes flow through parallel porous plates. The liquid flow was withdrawn through the porous medium walls under controlled uniform flow of rate. Exploration of Micro-polar fluid flow was done in [6] where the fluid passed through Riga plates inclined at an angle $\alpha$ from the vertical plane. [13] established that velocity profile increases rapidly as inclination angle increases having used the Crank-Nicolson finite difference method to solve the governing equations. [4] performed an approximation technique on MHD fluid past an inclined plate in presence of Chemical reaction and radiation absorption. They established that the Hartmann number, inclined magnetic parameter, porosity, and angle of inclination leads to a declination of the velocity. Similarly,[2] Chandrawat et al, performed a numerical simulation where the considered plates were inclined and porous. More studies on creeping flow were done by Ismail in [8]. Similarity transformation was employed to solve the equations and analytically presented on graphs. It was established that the density and number of dust particles lower the velocity of the fluid. It was also noted that dust particles move at higher velocity than fluid particles.

M. R. Islama et al [7], studied the flow of Dusty Fluid Past Riga Plates. Unlike in [7] where the magnetic field was not inclined at various angles, this study seeks to establish the effect of inclining Riga plates on the velocity profiles of a fluid.

Investigations on creeping flow has not exhaustively been made. For instance, Flow through inclined parallel Riga plates and effect of dust on fluid velocity. This study will therefore seek to bridge this knowledge gap by investigating the effect of inclining riga plates and dusty particle concentration on unsteady MHD Stokes flow of a dusty fluid.

## 2. Problem formulation

Consider a flow of a dusty fluid between parallel Riga plates. The plates are inclined at angle $\eta$ from the horizontal plane, stationary and at constant but different temperatures. The fluid is taken to be viscous and incompressible. The flow is assumed to be along the x-axis while y-axis is perpendicular to the flow. The fluid flow is due to pressure gradient (p). The presence of Riga plates influence Uniform magnetic force on the fluid. The velocity of the fluid is taken to be $u$ and that dusty fluid taken to be $u_d$. The following is a sketch of the physical concept of the problem. Where; g is the force of gravity,$B_o$ is the inclined magnetic field,$\eta$ is the angle of inclination of the Riga plates, The

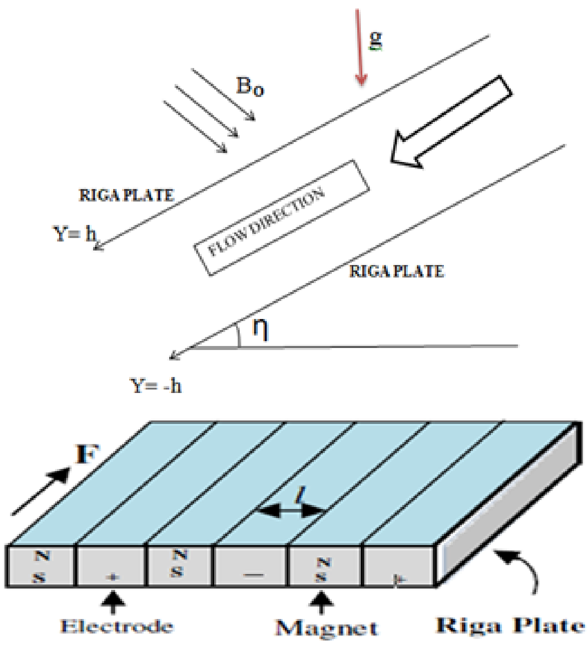



**Fig. 1.** Geometry of the physical problem

developed governing equations are;

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\pi J_0 M_0}{8\rho} e^{\frac{-y\pi}{l}} + g\sin\eta - \frac{u\sigma}{\rho}\vec{B}_o^2\sin^2\gamma - \frac{\nu u}{k_*} - \frac{1}{\rho}K_s N_d(u-u_d) \tag{1}$$

$$m_d \frac{\partial u_d}{\partial t} = K_s N_d (u - u_d) \tag{2}$$

where; $u$ is the velocity of the fluid particle, $p$ is the pressure of the flow, $J_o$ is the current concentration/density in the Riga plates electrodes. $M_o$ is the magnetic field effect created by the alternating permanent magnets on Riga plate and $l$ is the distance between the electrodes, which is the width of the magnets, $\rho$ is the density of the fluid, electrical conductivity feature of the fluid is $\sigma$, $\gamma$ is the angle of inclination of the magnetic field, $k_*$ is the porous medium permeability dimension, $K_s$ is the Stokes constant, $N_d$ is the number of dust particles per unit volume, $m_d$ is the mass of dust particles,

### 2.1. Non-Dimensionalisation of the governing equation

The main objective of Non-dimensionalisation is to set the solutions obtained in situation of a given set of conditions to be applicable to a geometrically similar environment but experiencing totally different conditions.
The following parameters are used to non-dimensionalise the governing equations.

$$\hat{x} = \frac{\pi}{l} x,\ \hat{y} = \frac{\pi}{l} y,\ \hat{u} = \frac{l}{\pi v} u,\ \hat{u_d} = \frac{l}{\pi v} u_d,\ \hat{p} = \frac{l^2 p}{\pi^2 \rho v^2},\ \hat{t} = \frac{\pi^2 v}{l^2} t. \tag{3}$$

where the Pertinent parameters generated include;

$$\begin{aligned}
&\text{Pressure Gradient:} && -\frac{\partial \hat{p}}{\partial x} = C\\
&\text{Modified Hartman Number:} && \frac{l^3 J_o M_o}{8\rho\pi^2 v^2} = H_r\\
&\text{Fluid Concentration Parameter:} && \frac{l^2 K_s N_d}{\rho \pi^2 v} = G\\
&\text{Eckert Number:} && \frac{\pi^2 v^2}{l^2 C_p (T_2 - T_1)} = ec\\
&\text{Prandtl Number:} && \frac{\rho C_p v}{k_c} = P_r\\
&\text{Particle Mass Parameter:} && \frac{M_d v \pi^2}{N_d K l^2} = S\\
&\text{Permeability Parameter:} && \frac{l^2}{k_* \pi^2} = L\\
&\text{Gravity Parameter:} && R = f \sin\eta \text{ where } f = \frac{g l^3}{\pi^3 v^3}\\
&\text{Joule Parameter:} && \frac{l \sigma B_0^2 \sin^2\gamma}{\rho c_p \pi (T_2 - T_1)} = M\\
&\text{Flow direction constant:} && \frac{\nu}{v} = \xi
\end{aligned} \tag{4}$$

substituting the above parameters in the equations (1) and (2) gives;

$$\frac{\partial \hat{u}}{\partial \hat{t}} = C + \xi \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + H_r e^{-\hat{y}} + R - \hat{u}(lM - L) - G(\hat{u} - \hat{u_d}) \tag{5}$$

$$\frac{\partial \hat{u_d}}{\partial \hat{t}} = S(\hat{u} - \hat{u_d}) \tag{6}$$

## 3. Solution of the problem

To solve the set of equations(5) and (6), explicit Finite difference method is used. This is because the method satisfies basic features of consistency, convergence and stability.

The length of the Riga plate is taken to be $x_{max} = 10$ cm which imply that $x$ varies from 0 to 10 and the distance between the plates is 2cm since the $y_{min} = -1$ and $y_{max} = 1$.
The horizontal axis(space variable) is sectioned into (N+1) intervals of equal $\Delta$y length that is indexed by $j = 0, 1, ....N$

and the vertical axis(time variable) divided into (M+1) intervals of length Δt indexed by $k = 0, 1, ...M$. The computations uses space and time points (j and k). The descretised form of equation (5) and (6) to solve for $u$ and $u_d$ at each grid point are presented below:

$$u_j^{k+1} = \left(C + \xi \frac{u_{j+1}^k - 2u_j^k + u_{j-1}^k}{\Delta y^2} + R - u_j^k(lM - L) + H_r e^{-y} - G(u_j^k - (u_d)_j^k)\right)\Delta t + u_j^k \tag{7}$$

$$(u_d)_j^{k+1} = S\Delta t\,(u_j^k - (u_d)_j^k) + (u_d)_j^k \tag{8}$$

The two equations are subjected to the following boundary conditions.

$$\hat{u} = 0,\ \hat{u_d} = 0,\ at\ y = -h \tag{9}$$
$$\hat{u} = 0,\ \hat{u_d} = 0,\ at\ y = h$$

## 4. Results and Discussion

The effects of Inclining Riga plates and dusty particles on velocity in an inclined magnetic field is studied. The various parameters such as pressure gradient, Modified Hartman number among others are varied and their effect on the obtained results of the equations discussed as well. The continuous line displays the fluid particle distribution while the dotted lines indicate the flow of dust particles. The velocity profiles are investigated having fixed C = 2, $\xi$= 1, G = 0.8, Ec = 0.01, $P_r$ = 0.71, Q = 0.8, L = 0.5, S = 0.5, R = 9.5, M = 0.5, $H_r$=1

### 4.1. Effect of dust particles concentration (G) on velocity profile.

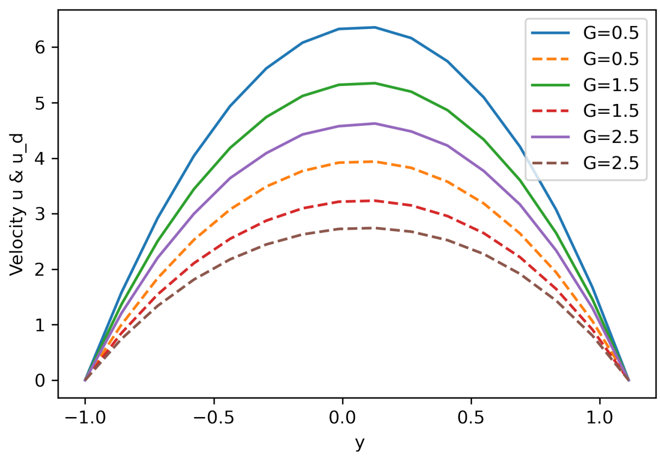



**Fig. 2.** Effect of G on u and $u_d$

The influence of dust particles concentration on a velocity of clean fluid is illustrated by fig. 2 above. An increase in G results in decrease in velocity for both Fluid and dust particles' velocity profiles. This is because the fluids viscosity increases and hence decelerating the flow behavior.

### 4.2. Effect of inclining Riga plates on velocity profile

For simplicity $f = 1$ which allows for the variation of the Riga plate inclination angle $\eta$ as tabulated below. The velocity profiles for both fluid and Dust particles are observed and noted at constant time $t = 2.5$. When the array corresponding to the variation in the inclined angle $\eta$ is plotted, then the fig. 3 below is obtained.

It can be seen from the visualized fig. 3 that velocity profiles are affected by the changes in the angle of inclination of the Riga plates. As the angle increases, Fluid flow through the plates increases as well. The velocity of dust particle is low compared to that of fluid particle.

**Table 1.** Angle $\eta$ with corresponding values of R and velocity profile

| ANGLE ($\eta^{\circ}$) | R=fsin$(\eta)^{\circ}$ | Fluid velocity ($u$) | Dust velocity ($u_d$) |
|---|---|---|---|
| 15° | 0.25882 | 0.72414616 | 0.49947165 |
| 30° | 0.50000 | 0.77956224 | 0.53797514 |
| 45° | 0.70711 | 0.82715003 | 0.57103948 |
| 60° | 0.86603 | 0.86366517 | 0.59641046 |
| 75° | 0.96593 | 0.88661926 | 0.61235913 |
| e 90° | 1.00000 | 0.89444754 | 0.61779828 |

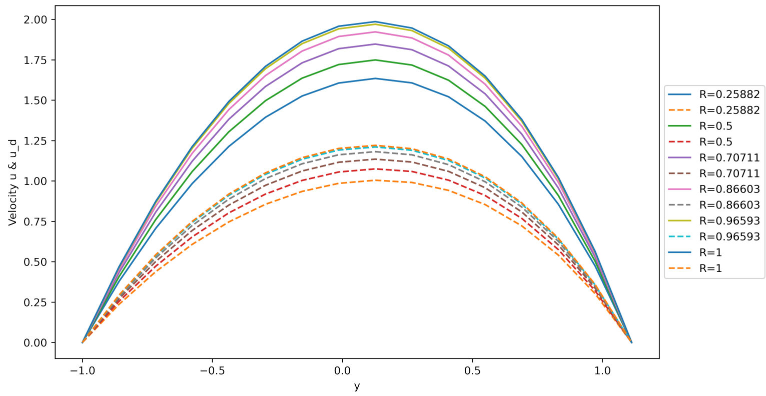



**Fig. 3.** velocity u and $u_d$

## 4.3. Effect of changes in inclination of field of magnetism on velocity

From $\sin^2\gamma = M$, tabulating the values of M, the trend of $u$ and $u_d$ can be observed.

**Table 2.** Angle $\gamma$ with corresponding values of M and velocity profile

| ANGLE ($\gamma^c irc$) | M=$\sin^2(\gamma)^{\circ}$ | Fluid velocity ($u$) | Dust velocity ($u_d$) |
|---|---|---|---|
| 0° | 0.00000 | 1.85110597 | 1.2586504 |
| 15° | 0.06699 | 1.80150129 | 1.22985932 |
| 30° | 0.25000 | 1.67908695 | 1.15783212 |
| 45° | 0.50000 | 1.53761442 | 1.07278264 |
| 60° | 0.75000 | 1.41944846 | 1.00018727 |
| 75° | 0.93301 | 1.34465207 | 0.95348336 |
| 90° | 1.00000 | 1.31937206 | 0.93756613 |

Table 2 displays the effect that field of magnetism has when varied from $0^0$ to $90^0$. From the table, both $u$ and $u_d$ are seen to decline from 1.85110597 to 1.31937206 and from 1.2586504 to 0.93756613 respectively. The diagram shows a significant drop of 0.172019 in velocity when the magnetic field adjusts from $0^0$ to $30^0$ and 0.1000764 when the magnetic field adjust from $60^0$ to $90^0$. The Magnetic field act as drag force on the fluid due to the interaction between magnetic field and the moving charges in the fluid hence opposing motion

## 4.4. Impact of pertinent parameters on velocity

Figure 5(a) displays the effect of time on the velocity profiles of both fluid and dust particles respectively.Its observed that fluid particle moves at a higher velocity as compared to dust particles.

Figure 5(b) visualizes the impact of pressure gradient C on velocity. An increase in pressure gradient leads to an increase in velocity for both dust and fluid particles.

Figure 5(c) indicates the variation of Modified Hartman number $H_r$ on velocity profiles. Both $u$ and $u_d$ increases as $H_r$ increases.

The effect of permeability on the flow is displayed by fig. 5(d). As L increases from 0.5 to 2.5, a sharp rise in $u$ and $u_d$ is noted.

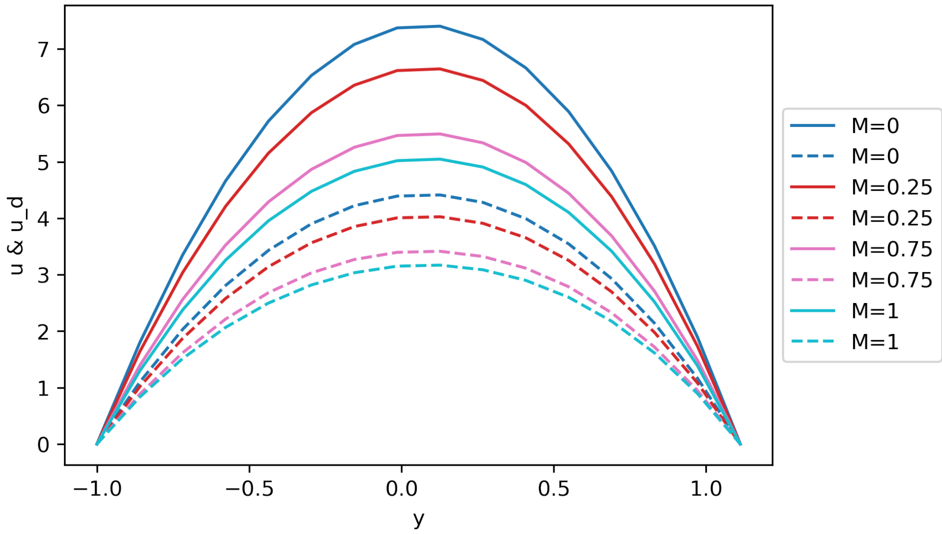
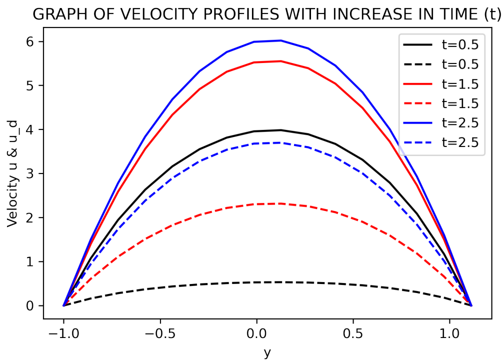
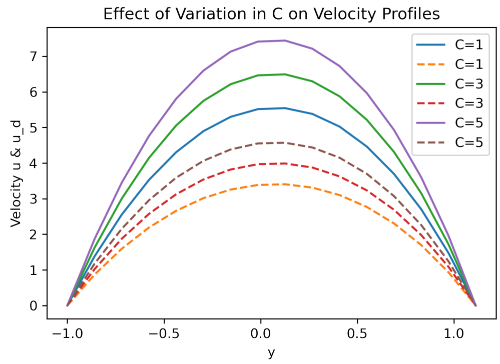


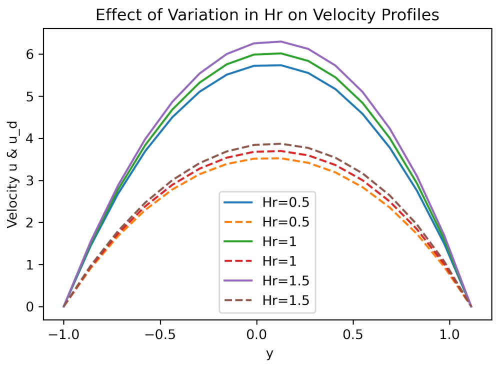

**Fig. 4.** velocity u and $u_d$



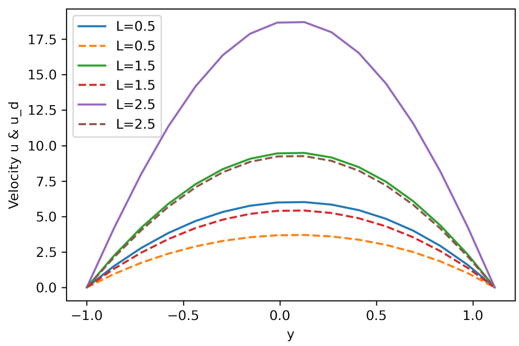

(a) Effect of time (t) on $u$ and $u_d$




(b) Effect of C on $u$ and $u_d$




(c) Effect of $H_r$ on $u$ and $u_d$




(d) Effect of L on $u$ and $u_d$

**Fig. 5.** Impact of various parameters on velocity profiles

## 5. Conclusion

The findings of this study indicates that:

- The velocity of the fluid particle $u$ is higher than that of dust particle $u_d$ as time is increases
- The concentration of dust particles G in a flow slows down the velocity of both dust and fluid particles.This is because G increases viscosity in Fluid flow.

- The angle of inclination has been observed to affect the flow rate of both fluid and dust particles. It is seen from the table that as the angle increases, Both $u$ and $u_d$ increases as well. Figure 4 presents the changes graphically.
- The velocities $u$ and $u_d$ are increased by increase in C, $H_r$, L, while they decreases with increase in M.

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